UNIVERSITY OF BRISTOL

School of Mathematics
Postgraduate Study in

Pure Mathematics

Contents

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1 Introduction

The Pure Mathematics group in the School of Mathematics has an international reputation for its research work. The main research themes lie in algebra (representation theory), in analysis (especially partial differential equations, potential theory and spectral theory), in set theory and mathematical logic, in number theory (analytic number theory and arithmetic combinatorics) and in ergodic theory and dynamical systems. Postgraduate students may study in any one of these areas towards the degrees of Ph.D. and M.Sc. by research.

Students whose undergraduate degree courses have not included sufficient preparatory material for postgraduate study with the group are encouraged to take a certain number of postgraduate courses and their progression will depend on this (see Section 7).

2 Research in Analysis and Partial Differential Equations

Spectral Geometry and Partial Differential Equations: Michiel van den Berg

A classical theorem of Faber and Krahn states that among all open sets Ω in Euclidean space $\mathbb{R}^m, m = 2, 3, \cdots$ with volume $|\Omega| = 1$, the open ball with volume 1 minimizes the principle eigenvalue of the Dirichlet Laplacian. The proof relies on the Schwarz or spherical decreasing rearrangement of functions in the Sobolev space $H_0^1(\Omega)$ [1]. By rearranging the level sets of the two nodal domains of the second Dirichlet eigenfunction on Ω one shows that if the second eigenvalue of the Dirichlet Laplacian is minimised over all open sets with volume 1, the minimiser is the union of two disjoint balls with volume 1/2 each. It is not hard to show that all minimisers are of such type [1]. It is not known whether there exists an open minimiser for the third eigenvalue of the Dirichlet Laplacian over all open sets with volume 1. It has been conjectured that if this minimiser exists then it should be a ball with volume 1 if $m = 2, 3$, and the union of three disjoint balls with volume 1/3 each if $m = 4, 5, \ldots$. Recently it has been shown [2] that if $\Omega_{m,k}$ is a minimiser of the variational problem

$$\inf\{\lambda_k(\Omega) : \Omega \text{ open in } \mathbb{R}^m,\ |\Omega| < \infty, \mathbb{H}^{m-1}(\partial\Omega) \leq 1\},$$

and if $k \leq m + 1$ then $\Omega_{m,k}$ has at most four components. $\mathbb{H}^{m-1}(\partial\Omega)$ denotes the $(m-1)$ - dimensional Hausdorff measure of the boundary of Ω.

My current research is on variational problems and conjectures of the type described above, and on related problems in spectral geometry. I am happy to discuss possible projects by e mail.

References

[1] A.Henrot, *Extremum problems for eigenvalues of elliptic operators*, Frontiers in Mathematics, Birkhäuser, Basel (2006).

[2] M. van den Berg, M. Iversen, *On the minimization of the Dirichlet eigenvalues of the Laplace operator*, arXiv:0905.4812v1.

Function Spaces, Spectral Theory and Exceptional Sets: Yuri Netrusov

My main area of research is the study of spaces of weakly differentiable functions defined on open subsets of Euclidean space $\mathbb{R}^n$. Let Ω be a domain in $\mathbb{R}^n$, and let $1 \leq p \leq \infty$. Denote by $L^{l,p}(\Omega)$ the Sobolev space equipped with the following semi-norm $(\int_\Omega |\nabla^l f(x)|^p dx)^{1/p}$. One of the most interesting problems connected with these spaces is the description of their traces on subsets of Ω (the case $l = 1$, $p = 2$ is very important for applications to partial differential equations and harmonic analysis). This problem was investigated by many analysts but the general situation is still unclear.

Many problems of spectral theory can be reformulated in terms of Sobolev spaces. For instance one can see that eigenvalues of Neumann Laplacian coincide with s-numbers of imbedding

$$L^{1,2}(\Omega) \subset L^2(\Omega).$$

Suppose that X is some space of functions defined on $\mathbb{R}^n$. Let K be a compact subset of $\mathbb{R}^d$. Denote by $Cap_X(K) = inf||g||_X$ capacity associated with this space. Here the infimum is taken over all functions g such that $g = 1$ on the set K, $g \in C_0^\infty(\mathbb{R}^n)$. One can see that in the case when X is coincides with the Sobolev space $L^{l,1}$, $l < d$ or $L^{1,2}$ the quantity Cap_X is equivalent to the Hausdorff content or harmonic capacity respectively. There are many interesting questions connected with these quantities for different functional spaces. Most attention is paid to the class of all compacts K such that $Cap_X(K) = 0$. These classes appear in many problems of analysis.

Discrete Geometry, Geometric Measure Theory, Mathematical Mechanics and Hamiltonian Chaos: Misha Rudnev

The main theme of my current research is the interaction of harmonic analysis, geometric combinatorics, geometric measure theory and analytic number theory, in particular various manifestations of the notion of curvature therein. There are a number of fundamental questions arising at the crossroads of these areas of mathematics. These questions have lead to conjectures which today are wide open and mysteriously intertwined. Around these conjectures, there is a constant supply of exciting new problems which appeal to a variety of mathematical taste and expertise.

Some problems in geometric combinatorics and measure theory

My recent efforts in geometric combinatorics and measure theory are centered around the Erdös/Falconer distance conjectures which ask, in a variety of settings, for the smallest number of distances determined by subsets of the Euclidean space. Namely, let $E \subset \mathbb{R}^d$. Define its *distance set* as

$$\Delta(E) = \{\|x - y\|, \ \ x, y \in E\}.$$

How big does E have to be so that its distance set is conspicuous? What I mean by that precisely is – suppose E is a discrete set of $|E| \gg 1$ elements and let $q \gg 1$ be a large real. What is the minimum cardinality of E to guarantee that the number of elements $|\Delta(E)|$ in the distance set is at least q? Paul Erdös conjectured in 1946 that $|\Delta(E)| \geq q$, whenever $|E|$ is asymptotically greater than $q^{\frac{d}{2}}$. More precisely, for any $\varepsilon > 0$ there exists a constant C_ε (depending also on the dimension d, but not on E) such that

$$|\Delta(E)| \geq q, \quad \text{whenever} \quad |E| \geq C_\varepsilon q^{\frac{d}{2}+\varepsilon}.$$

This conjecture is nowhere near resolution. As of today, the best result in dimension 2 (which in fact may be the most difficult dimension: there is some not totally unfounded hope that in $d \geq 4$ the matters may become easier) is due to N. Katz and G. Tardos, saying that $|\Delta(E)| \geq C|E|^{\approx .86}$ for some C. The result is obtained by methods of combinatorial geometry and rests on a repeated application of the Szemerédi-Trotter incidence theorem. The latter theorem says that if one has m lines and n points in the plane, the total number of incidences I, i.e. pairs (p, l) such that a point p lies on the line l, is bounded from above as follows:

$$I \leq 5[m + n + (mn)^{\frac{2}{3}}].$$

The continuous analogue of the Erdös distance conjecture is known as the Falconer distance problem. Suppose now that E is a compact Borel set in $\mathbb{R}^d$. It is conjectured that $\Delta(E)$ has *positive Lebesgue measure*, whenever the Hausdorff dimension of E exceeds $\frac{d}{2}$. The fact that there is the same $\frac{d}{2}$ in both discrete and continuous problems (even if you do not quite know what the Hausdorff dimension is) is certainly not an accident! Falconer himself proved in 1986 that $\Delta(E)$ has positive Lebesgue measure, whenever the Hausdorff dimension of E exceeds $\frac{d+1}{2}$. The Falconer conjecture has been studied by methods of harmonic analysis, and the closest to $\frac{d}{2}$ one could get so far is $\frac{d+1}{3}$, due to T. Wolff and M. B. Erdoğan.

The distance conjectures and related problems can be studied from the analytic, number theoretic and combinatorial viewpoint. For example, there is a link between the Fourier transform estimates related to the Falconer distance problem and the celebrated Freiman theorem in additive number theory. The latter theorem says that if a set of numbers $A \subset \mathbb{Z}$ has a property that $|A + A| \leq C|A|$ (i.e. the set of possible pair-wise sums of the elements of A does not exceed A in cardinality by more than a constant factor C), then A is contained in an arithmetic progression with at most $k(C)$ generators. (Freiman's theorem has been recently generalised by B. Green and I. Ruzsa to the case of an arbitrary Abelian group instead of $\mathbb{Z}$.)

From the analytical point of view, it would not matter if the distance $\|\cdot\|$ in the definition of the distance set became anisotropic, i.e. instead of the unit sphere S^{d-1} one dealt with the boundary ∂K of some smooth well-curved body K. Studying K-distances is fruitful for at least one good reason: they provide a rich source of examples and counter-examples. The approach of T. Wolff and M. B. Erdoğan, vindicating the dimensions greater than $\frac{d+1}{3}$ for the Falconer distance problem target best possible point-wise estimates for the "spherical average" $\sigma_\mu(r) = \int_{S^{d-1}} |\widehat{\mu}(r\omega)|^2 d\omega$, where μ is a Borel measure, supported on the set E. (In order that $\Delta(E)$ have positive Lebesgue measure it suffices that the so-called *Mattila integral* $M_\mu = \int_1^\infty \sigma_\mu^2(r)\, r^{d-1} dr$ converge.) Best possible point-wise estimates for the spherical average $\sigma_\mu(r)$ are available in $d = 2$ (where they alone do not suffice to resolve the Falconer conjecture), however in dimensions $d \geq 4$ they are not known, but can in principle turn out to be in a sense more regular than for $d = 2$. In $d \geq 4$ these estimates alone could possibly resolve the Falconer conjecture. This question does not appear to be totally unfathomable within the harmonic analysis approach. Interestingly enough, due to a recent work of myself and A. Iosevich, its positive resolution would entail that there exist no well-curved smooth convex bodies K in higher dimensions, such that for no matter how large t, the boundary ∂K blown up t times had more than integer lattice points, for any $\varepsilon > 0$. This is a classical analytic number theory fact if K is a sphere.

There is also a connection between the asymptotic behaviour of Fourier transforms of measures in $\mathbb{R}^d$ and properties of a class of Diophantine equations. In the context of the finite field analog of the Erdös/Falconer distance problem, there is a connection between this problem and asymptotic properties of classical number-theoretic Kloosterman sums and its "angles".

Finally, it is not understood, what is the proper niche, occupied by the Erdoös/Falconer distance problem amidst other giant open questions in harmonic analysis, such as first of all the Restriction/Kakeya problem (claiming that the Hausdorff dimension of a set in $\mathbb{R}^d$, containing a unit segment in each direction is d). To this effect, it seems very sensible to study the Falconer problem on the sphere S^{d-1} and see how the curvature comes into play. It does not seem unreasonable that the resolution of the Falconer conjecture on S^2 is strongly correlated with the Restriction conjecture in $\mathbb{R}^3$, but the precise connections are yet to be discovered.

If you are interested in any of these questions, please do not hesitate to e-mail me at `m.rudnev@bris.ac.uk`. You can look up my recent work on these problems on
`http://www.maths.bris.ac.uk/∼maxmr/papers.html`

Here is a list of excellent on-line notes which provide more than the introduction to harmonic analysis and adjacent areas of mathematics (see also the links on these pages).

- Ben Green's notes on Restriction and Kakeya
  `http://www.maths.bris.ac.uk/∼mabjg/rkp.html`
- Expository notes by Alex Iosevich
  `http://www.math.missouri.edu/∼iosevich/expositorypapers.html`
- Thomas Wolff's Lectures on Harmonic Analysis, edited by Izabella Łaba
  `http://www.math.ubc.ca/∼ilaba/wolff`
- Terry Tao's Harmonic Analysis page
  `http://www.math.ucla.edu/∼tao/harmonic/`

  I am happy to discuss further details of possible projects in this area
  by email: m.rudnev@bristol.ac.uk.

3 Research in Number Theory

Number Theory:
Tim Browning

I am interested in problems that lie at the interface of analytic number theory and Diophantine geometry. Often this involves studying aspects of the locus of rational or integral solutions to a system of Diophantine equations. Such a system defines an algebraic variety, and one might then ask to what extent the behaviour of this locus is dictated by the geometry of the variety. At other times one wants to study problems coming from analytic number theory. Many such problems refer to (or can be reduced to) questions involving rational or integral solutions to Diophantine equations. This area of research combines tools from analytic number theory with concepts from algebraic geometry, and would suit anyone who has an interest in both of these two fields.

I am happy to discuss further details of possible projects in this area by email, some indication of which can also be found on my web page:
`http://www.maths.bris.ac.uk/∼matdb/`.

Algebraic Number Theory and Elliptic Curves:
Tim Dokchitser

I mostly work in algebraic number theory and elliptic curves, and especially with things related to Birch-Swinnerton-Dyer Conjecture and arithmetic of L-functions. I am also interested in all kinds of connections between number theory and Galois theory, finite groups and representation theory. Most of the things that I do rely heavily on computers and computational algebra systems. I use computer experiments a lot to formulate and to test conjectures, and to get inspiration about their proofs as well.

I have many possible projects that concern elliptic or hyperelliptic curves, abelian varieties and their L-functions; feel free to drop by to discuss them if you are interested. You can also have a look on my webpage,

`www.maths.bris.ac.uk/~matyd/`

for my publications, summer school lecture notes and the like.

Analytic Number Theory and Diophantine Approximation:
Alan Haynes

My research is motivated by problems in analytic number theory (e.g. the distribution of prime numbers) and Diophantine approximation (approximating real numbers by rationals). Both of these subjects are well connected with most branches of mathematics, and in recent years I have been exploring connections with dynamical systems (ergodic theory), probability theory, and harmonic analysis. The goal of my current research is to develop tools and techniques from these areas with a view toward proving results in number theory. This is a fast growing region of mathematical research and there are a lot of interesting open problems. Some examples of possible PhD projects are:

1. Develop the theory of Diophantine approximation in solenoids (i.e. mathematical solenoids) and Adelic spaces over a number field, and explore potential applications to theoretical physics. This is a broad topic which would allow you to draw from and incorporate a large background of ideas from many places.

2. Investigate the use of martingales and extremal functions from Fourier analysis in the construction of low-discrepancy sequences in higher dimensions. This is bordering on an important unsolved problem which has applications to numerical simulation in the sciences.

3. Investigate recent problems and results about the dynamics of circle rotations and their generalizations. For example, is there a times-2 invariant measure supported on the set of badly approximable numbers? There are a lot of questions here which have direct consequences for open problems in number theory.

These examples should give you the flavour for the types of problems I am interested in. I am also happy to discuss other ideas you may have for potential projects.

Number Theory:
Lynne Walling

I work with quadratic forms and various types of automorphic forms, which are nicely behaved functions whose Fourier coefficients encode number theoretic information. One type of automorphic form, called theta series, encodes the number of times a quadratic form represents each integer, or more generally, each quadratic form of smaller dimension. So understanding these automorphic forms helps us understand these representation numbers. On the other hand, the arithmetic theory of quadratic forms helps us understand automorphic forms, particularly those called Siegel modular forms. While most researchers study automorphic forms using analytic or representation theoretic tools, I mainly use algebraic tools.

I am happy to discuss further details of possible projects in this area by email: l.walling@bristol.ac.uk.

Analytic Number Theory:
Trevor Wooley

My research is centred on the Hardy-Littlewood (circle) method, a method based on the use of Fourier series that delivers asymptotic formulae for counting functions associated with arithmetic problems. In the 21st Century, this method has become immersed in a turbulent mix of ideas on the interface of Diophantine equations and inequalities, arithmetic geometry, harmonic analysis and ergodic theory, and arithmetic combinatorics. Perhaps the most appropriate brief summary is therefore "arithmetic harmonic analysis".

Much of my work hitherto has focused on Waring's problem (representing positive integers as sums of powers of positive integers), and on the proof of local-to-global principles for systems of diagonal Diophantine equations and beyond. More recently, I have explored the consequences for the circle method of Gowers' higher uniformity norms, the use of arithmetic descent, and function field variants. The ideas underlying each of these new frontiers seem to offer viable approaches to tackling Diophantine problems known to violate the Hasse principle.

I am happy to discuss possible research projects by email — anything connected with the topics above (arithmetic harmonic analysis). See my web page for some surveys and other details of work with which I am currently involved. (http://www.maths.bris.ac.uk/∼matdw/)

4 Research in Ergodic Theory and Dynamical Systems

Dynamical Systems and Number Theory: Alex Gorodnik

My research interests are in ergodic theory, theory of transformation groups, and number theory. While most nontrivial dynamical systems exhibit complicated chaotic behaviour, it is possible to develop techniques to analyse asymptotic properties of the orbits. This is not only of fundamental importance in the theory of dynamical systems, but also has many far-reaching applications to number theory. For instance, these techniques can be used to study solutions of Diophantine equations and inequalities. Another important scheme in my research is various rigidity phenomena in dynamical systems. Given a dynamical system, one may ask to describe, for instance, maps that commute with the action, closed invariant sets, invariant measures, etc. It turns out that in several important cases these objects have algebraic origin and can be completely classified, which leads to many surprising applications.

I will be happy to supervise a PhD project in one of the following directions:

- statistical properties of orbits for actions of large groups,
- dynamical systems appearing in Diophantine geometry,
- rigidity properties of dynamical systems.

Prospective students should consult a gentle introduction to my research:

http://www.maths.bris.ac.uk/$\sim$mazag/res.html

and contact me in person or by email to discuss details of potential projects.

Dimension Theory in Dynamical Systems: Thomas Jordan

Properties of self-similar sets

A self-similar set is given by the attractor of an iterated function system consisting of a finite number of contracting similarities. It is well know that if the system satisfies the open set condition then the Hausdorff dimension is given by the solution to $\sum r_i^s = 1$ (where r_i are the contraction ratios). However in some cases this still gives the dimension even when the open set condition is not satisfied. A potential problem is to find a condition about the similarities which is equivalent to the Hausdorff dimension dropping below s. A further area of study could be when is it possible for self-similar sets with positive Lebesgue measure to have empty interior. In, [1] it is shown that such sets can exist in R^d for $d \geq 2$. However whether such sets exist in 1-dimension is still unknown. A more tractable problem may be to look at the same problem for random self-similar sets in 1 dimension.

References

[1] M. Csörnyei, T. Jordan; M. Pollicott; D. Preiss; B. Solomyak. Positive-measure self-similar sets without interior. Ergodic Theory Dynam. Systems 26 (2006), no. 3, 755–758

Dimension theory of non-conformal systems

The properties of self-similar sets satisfying suitable separation condition is now very well understood. Similarly if we take a non-linear family where the maps are conformal (at every point the derivative is a similarity) then the same is generally true. However if we consider the case of self-affine system or non-conformal nonlinear systems then much less is known. There are several possible projects in this area. For example there is a formula for the dimension of self-affine systems which holds almost surely. However there are very few cases where it can be shown to hold for a specific example particularly if the dimension is greater than one. It would be of great interest to find more cases where this can be shown. For non-linear systems an interesting project would be to see if it is possible to find the dimension under suitable random perturbations (this can be done for self-affine sets).

If you are interested in either of the above projects or in any project in the dimension theory of dynamical systems or fractal geometry then you are welcome to contact me by email (thomas.jordan@bris.ac.uk).

Number theory. Quantum Chaos; Dynamical Systems: Jens Marklof

Quantum maps

Quantum maps form some of the simplest models of quantum chaos. Quantum maps are certain unitary N by N matrices, whose eigenvalues and eigenvectors exhibit, in the limit of large N, the same statistical features as the energy levels and stationary states of more realistic quantum systems. One of the hot topics is currently the analysis of localization properties of the eigenstates [1], and the analysis of so-called intermediate statistics in pseudo-integrable systems [2].

References

J. Marklof and S. O'Keefe, Weyl's law and quantum ergodicity for maps with divided phase space, with an appendix by S. Zelditch, *Nonlinearity* **18** (2005) 277-304

O. Giraud, J. Marklof and S. O'Keefe, Intermediate statistics in quantum maps, *Journal of Physics A* **37** (2004) L303-L311

Almost modular functions and distribution modulo one

I have recently discovered a novel class of functions, the almost modular functions [1], or AMFs. Classical modular functions are among the most important objects of study in number theory. They are defined as functions that are invariant under the action of the modular group or one of its congruence subgroups. On the other hand, an almost modular function is a function which is approximately invariant, in that the error in the approximation tends to zero as one considers modular subgroups of increasing index. The main outcome of my investigation is that AMFs satisfy very similar limit theorems as modular functions. An important example of an AMF is the error term for distribution of fractional parts of the well studied number-theoretic sequence $x, 4x, 9x, 16x, \ldots$ It thus follows that the error term of this classical sequence satisfies a limit theorem, where the limiting distribution, perhaps surprisingly, deviates from a Gaussian law. There is a wealth of other examples of almost modular functions; holomorphic variants are for instance discussed in [2]. The research on AMFs is only at its beginning!

References

J. Marklof, Almost modular functions and the distribution of nx modulo one, *IMRN International Mathematics Research Notices* **39** (2003) 2131-2151

J. Marklof, Holomorphic almost modular forms, *Bulletin of the London Mathematical Society* **36** (2004) 647-655

Prerequisites are a strong background in pure mathematics, especially complex analysis, and elementary Mathematica, Maple, Matlab or Python skills.

Other projects

If you have a strong background in pure mathematics and are interested in problems in mathematical physics or number theory, you are most welcome to contact me at j.marklof@bristol.ac.uk to discuss other current research projects.

Ergodic Theory and Teichmüller Dynamics:
Corinna Ulcigrai

My main research interests are in ergodic theory and Teichmueller dynamics, an area of research in dynamical systems which has developed and bloomed in the last decades and that often involves arithmetical, geometrical and combinatorial tools.

Examples of systems studied in Teichmueller dynamics are:

- Polygonal billiards, in which a point-particle moves in a planar polygon bouncing elastically at sides.
- Geodesics on translation surfaces, which are locally Euclidean surfaces but at some conical singularities.
- Maps of the interval which are piecewise isometries, called interval exchange transformations (IETs).

One is interested in investigating ergodic properties that describe how chaotic these systems are.

The properties of these elementary systems are beautifully and deeply connected with the dynamics on an abstract space of deformations, more precisely with the Teichmueller geodesic flow and the SL(2,R) action on moduli spaces. At the level of interval exchange transformations, this connections can be exploited at a more combinatorial level, using a continued fraction algorithm called Rauzy-Veech induction.

Some areas of research for possible Phd projects:

- Area preserving flows on surfaces: in a natural class of flows locally given by a Hamiltonian, I recently studied properties like mixing and weak mixing; many interesting questions about spectral properties are open;
- Cutting sequences: in a joint work with Smillie, we gave a characterization of the symbolic sequences which code linear trajectories in regular polygons. Similar questions could be addressed in other translation surfaces with the lattice property.
- Interval exchange transformations: questions about distributions of orbit points, as spacings and discrepancy, in the spirit of limit theorems or for special classes of IETs (like bounded type).

If you are interested in knowing more about this research area and potential projects, feel free to contact me. My email address is corinna.ulcigrai@bristol.ac.uk

5 Research in Algebra

Representation Theory:
Jeremy Rickard and Aidan Schofield

Representation theory is a field that covers many of the currently most exciting areas of algebra, and at present, there are two permanent members of the Bristol School of mathematics, Professors Aidan Schofield and Jeremy Rickard, active in research on various aspects of this field.

The two main areas they work in are representations of quivers (for a definition, see below) and representations of finite groups. Both of these can be thought of as special cases of the study of modules for a ring: in the case of quivers, the ring in question is the so-called 'path algebra' of the quiver and in the case of groups it is the group algebra.

Aidan Schofield works on the representation of quivers and related questions. A quiver is a set of vertices together with a set of arrows between these vertices, and a representation of a quiver is just a way of assigning a vector space to each vertex of the quiver, and a linear map to each arrow. To give a simple example: the quiver might have a single vertex and a single arrow going from that vertex to itself; in this case a representation is just a vector space together with a linear endomorphism. The study of this rather simple idea of a quiver turns out to be fundamental to representation theory and to other areas of mathematics because many classification problems in mathematics can be reduced to studying representations of a quiver. To return to the very simple example given earlier, the classification of representations of the quiver with one vertex and one arrow is equivalent to that of square matrices up to conjugacy, which is given by the Jordan normal form.

The techniques used in the study of representations of quivers involve geometric ideas. The basic idea is that if you fix the vector spaces involved in a representation of some quiver and fix a basis for each one, then the choice of linear maps just involves choosing the entries in a collection of matrices, and hence is equivalent to choosing a point in a (usually rather large) vector space V whose dimension is the total number of entries that need to be chosen. However, many choices will lead to isomorphic representations, as a different choice of basis might have been made for the vector spaces. A little thought shows that there is a group acting on V in such a way that elements of V give isomorphic representations precisely when they are in the same orbit. Thus we are led to the study of orbits of groups acting on vector spaces, which is the subject of geometric invariant theory. If you come to Bristol to do a Ph.D. in this subject, then, you should expect to learn a certain amount of algebraic geometry.

Jeremy Rickard works on the modular representation theory of finite groups. A representation of a finite group G is just an action of G by linear maps on a vector space V, or in other words a group homomorphism from G to the group $GL(V)$ of invertible linear endomorphisms of V. You may have come across representations over the complex numbers in an undergraduate course. In this case, the problem of classifying representations reduces to finding the (finitely many) irreducible representations, as a general representation is a sum of these. Moreover, one can calculate which irreducibles occur as constituents of a given representation from its character, so the character table of a group contains virtually all the information about its representation theory.

The word 'modular' refers to the study of representations over fields of non-zero characteristic p, where p divides the order of the group G. Here matters are much more complicated, as it is no longer true that every representation is a sum of irreducibles, and it is only in the simplest cases that a complete classification can be found (for example, there is not even a classification of representations of $C_3 \times C_3$ over a field of characteristic 3). However, this added complexity has the compensating advantage that more powerful techniques can be brought to bear.

This field is a very exciting one to be working in at present, as many deep and intriguing conjectures have arisen over the last decade or so, completely transforming the subject. If you come to Bristol to study modular representation theory, you will be working on questions intimately related to these exciting recent developments: although it might be over-ambitious to expect to settle the main problems that are currently motivating research in the area, there are many more tractable problems that they have given rise to.

Much of the most important current work on modular representation theory involves ideas related to homological algebra and algebraic topology. Topological ideas come into group representation theory in many different ways: one of the more obvious is that many important representations have geometric constructions – if a group acts on a topological space then it acts on the homology groups of the space, which therefore become representations of the group.

If you come to Bristol to do a Ph.D. in this subject, you will learn a great deal about the more algebraic side of algebraic topology.

6 Research in Logic and Set Theory

Set Theory:
Philip Welch

Modern set theory evolved from attempts by logicians to put the foundations of mathematical analysis on a sure axiomatic footing, and avoid the paradoxes of Russell and others. The Zermelo-Fraenkel ("ZF") axioms are commonly regarded as achieving this – but various questions remained unanswered about, for example, the *sizes* of subsets of the real line, the most notable being the Continuum Hypothesis. Since the 60's, and from the work of Gödel, we know such questions are unanswerable on the basis of ZF, even in theory. (Moreover whatever axiomatic theory we choose, there will be such unanswerable questions.) Set theorists study various "axioms of strong infinity" relating to ZF-models (structures that satisfy the ZF-axioms, much as a group satisfies the group axioms). A whole hierarchy of theories have been discovered, the study of which has evolved into a branch of pure mathematics in its own right. This study of "inner models" has used metamathematical methods to provide results in other areas of mathematics. It appears, somewhat paradoxically, that the presence of extremely large sets in the universe of all sets can affect how the analyst must view the real line for example.

At Bristol interest centres on several themes.

(i) Some current exciting developments relate *axioms of determinacy* (stated in terms of infinite 2-person perfect information games on integers: "do such games have winning strategies for either player?"), statements about the real line ("Is every uncountable set in (1-1) correspondence with the whole line?") and with large sets (such as so-called "measurable cardinals") which are startlingly remote from the sets of reals, but which involve such large cardinal numbers, and their inner models, generalising the work of Gödel. A program of research here at Bristol would typically involve a detailed investigation into the structural properties of such a model and how this model's existence effects those games' outcomes.

(ii) Combinatorics. Certain infinite cardinal properties lend themselves to characterisation in terms of *infinitary combinatorics*, generalising, for example the Ramsey Theorem on natural numbers. There are some difficult questions here, but there are numerous others that are suitable for tackling at the Ph.D. level.

(iii) In very weak systems of set theory (or even just the straightforward theory of analysis) only very simple games of the type mentioned in (i) can be determined. One can ask which kinds of axiom system are needed to prove the existence of winning strategies. It has recently been noticed that certain techniques used by philosophers when building *theories of truth* or *theories of definitions* use very similar systems. Certain kinds of abstract infinitary algorithm derived from computer science are also coincidentally of this kind. What are the exact relationships here?

7 Academic Staff in Pure Mathematics

Professor Michiel van den Berg:

Partial differential equations, in particular spectral geometry.

e-mail: m.vandenberg@bris.ac.uk

Dr Andrew Booker:

Analytic and algorithmic number theory.

email: andrew.booker@bris.ac.uk

Dr. Tim Browning:

Analytic number theory; Diophantine geometry.

email: t.d.browning@bris.ac.uk

Professor Brian Conrey:

Analytic number theory, especially the analytic theory of L-functions.

email: b.conrey@bris.ac.uk

Dr. Tim Dokchitser:

Algebraic number theory and elliptic curves.

email: b.conrey@bris.ac.uk

Dr Alex Gorodnik:

Dynamical systems and number theory

email: a.gorodnik@bris.ac.uk

Dr. Alan Haynes Analytic number theory and Diophantine approximation

email: alanhaynes@gmail.com

Dr Thomas Jordan :

Dimension theory in dynamical systems

e-mail: t.jordan@bris.ac.uk

Professor Jens Marklof:

Number theory; Quantum chaos; Dynamical systems.

email: j.marklof@bris.ac.uk

Dr. Yuri Netrusov:

Functional Spaces, Partial differential equations, Spectral theory.

email: y.netrusov@bris.ac.uk

Professor Jeremy Rickard:

Representation theory of finite-dimensional algebras. Modular representation theory of finite groups. Homological algebra.

email: j.rickard@bris.ac.uk

Professor Aidan Schofield:

Representation theory of finite dimensional algebras, Kac-Moody Lie algebras. Also invariant theory and the study of rings with polynomial identities.

email: aidan.schofield@bris.ac.uk

Dr Corinna Ulcigrai:

Ergodic theory and Teichmüller dynamics.

email: corinna.ulcigrai@bris.ac.uk

Dr Lynne Walling:

Number theory, working with quadratic forms and various types of automorphic forms.

email: l.walling@bris.ac.uk

Professor Philip Welch:

Set theory, fine structure and core models. Problems concerning determinacy, large cardinals and strong axioms of infinity.

email: p.welch@bris.ac.uk

Professor Trevor Wooley:

Analytic Number Theory, Diophantine Equations and Diophantine Problems, Harmonic Analysis. The Hardy-Littlewood Circle method, and the theory and applications of exponential sums.

email:trevor.wooley@bris.ac.uk

8 Ph. D and M.Sc. by Research

The normal entry requirement for a PhD in Bristol is a good honours degree (1st class or upper second class) in Mathematics or a related subject from a British University; equivalent qualifications from overseas institutions are accepted. Funded positions are awarded on a competitive basis. The funded period of study is three and a half years. The UK Research Councils currently award research studentships for maximum three and a half years for British and EU students who have been residents in UK for 3 years prior to application.

Each student has an adviser with whom he or she works. In many cases the adviser is chosen before the student arrives. This is necessary for certain studentships. In other cases, the adviser is chosen in the first few weeks of study. Many students are also supervised jointly by two advisers.

Research degrees are awarded on the recommendation of the examiners of the student's final dissertation; this includes an oral examination.

New starting students are expected to attend postgraduate lecture courses, and sometimes undergraduate lecture courses, in order to ensure a solid background knowledge of their subject. Weekly seminars keep the School abreast of current developments and all students are expected to attend. 'Formal' seminars are given by distinguished visitors from elsewhere in Britain and abroad. 'Informal' seminars are usually given by people already in Bristol, including postgraduates themselves.

Students are encouraged and supported financially to attend meetings and conferences relevant to their work. Normally students will be funded to attend at least one international conference during the tenure of their award, provided that they are presenting a talk about their work.

Masters programme

The School also has a **M.Sc. programme in Mathematical Sciences**. During this 1-year course students will focus on an area at the forefront of research in Mathematics where they will be able to attend specialised lectures and perform their own research under the supervision of a member of staff. This course runs for a full year from October to September. Students choose under the guidance of a mentor a combination of taught units that are usually examined in April and early June. Students will focus on their projects during the summer.

9 Studentships and Funding

British and EU Students

The main single source of funding for UK and EU students, who have been residents in the UK for 3 years prior to application, is the School's **EPSRC**-based Doctoral Training Account (DTA), which awards studentships on a competitive basis. Application must be made through the School, preferably by **1 April** for students wishing to start the following October. An EPSRC studentship meets all the tuition fees and pays a maintenance allowance to the student. The precise figures for each session are announced in mid-summer, but from October 2011 the value of this allowance is £13,590 plus fees per annum. No tax is payable on this income, nor is there any abatement of the allowance on account of income from other sources. Supplements are available for disabled students, mature students, students with dependents and for students with suitable postgraduate work experience.

Other students from European Union countries may be eligible for fees-only grants from the EPSRC and awards from the EU.

(There are no EPSRC awards available for students undertaking the M.Sc. by research). The awards, available for a maximum period of three and a half years, are made on a competitive basis and are generally only awarded to students with First or Upper Second Class Honors degrees or equivalent. Full details of the terms of EPSRC studentships may be obtained from:

Engineering and Physical Sciences Research Council
Polaris House
North Star Avenue
Swindon SN2 1ET
United Kingdom

or from the EPSRC website (`http://www.epsrc.ac.uk`).

These studentships are awarded competitively, that is to say to the best qualified students. Preference may be given to candidates who are taking up projects in certain 'earmarked areas' or changing university. In the recent past, candidates with first-class degrees and some good upper seconds have been awarded studentships.

The school should also have some studentships available for students from **UK** and particularly **EU** countries. These awards will typically cover fees and a maintanence allowance at the same level as the EPSRC (DTA)-studentships mentioned above. Information and

deadlines about such studentships will appear at
http://www.maths.bris.ac.uk/admin/jobs/
and
http://www.bris.ac.uk/studentfunding/eu.html

Further information is available from:

Student Funding Office
University of Bristol
Senate House (Ground floor)
Tyndall Ave
Bristol BS8 1TH
United Kingdom

Project Students

Sometimes a member of staff will have a PhD studentship for a specific project. These studentships will be available for UK and EU students. Any such studentships will be advertised at http://www.maths.bris.ac.uk/admin/jobs/.

CASE studentships

These are co-operative awards with an industrial partner, and so are more common in applied mathematics. They are awarded for specific topics proposed and applied for by the advisers.

Overseas Students

Overseas students should check
http://www.bris.ac.uk/studentfunding/overseas_pg/cent_overseas_schols.html
for news about sources of funding. Also each year the school may have some funded studentships that overseas students can apply for. Details of such studentships will appear on
http://www.maths.bris.ac.uk/admin/jobs/
A leaflet listing possible sources of funding for overseas students is available from:

Student Funding Office
University of Bristol
Senate House (Ground floor)
Tyndall Ave
Bristol BS8 1TH
United Kingdom

Tutorial, problem classes & marking work

Postgraduates usually have an opportunity to supplement their income by giving tutorials and marking undergraduate work. This supplement can often be quite significant. The hourly rate for this work is usually £13.79 and for tutorials this will include preparation time. Students are **not normally permitted** to give more than 6 hours of tutorials per week. Typically students have been earning in the range of £1600 to £1700 per year tax-free (these amounts are not guaranteed, but are typical of what students have been earning in the year 2008).

10 Enquiries and Application Procedures

Application Procedure

If you are interested in pursuing your postgraduate study in the school you should complete an online application, see

`http://www.bristol.ac.uk/prospectus/postgraduate/2011/intro/apply.html`.

Please include in the application the full list of courses and marks from your university, English language certificates for overseas students, two references and the areas of research you are interested in. These documents can be uploaded on the website. Letters of reference can also be sent directly by the reference writers. PhD applicants should already have, or expect to be awarded, a first or upper second class degree in a relevant subject. Submitting an application is in no way a commitment to accepting a place.

When the application has been received, UK applicants will be invited to visit Bristol to meet relevant staff. Reasonable travel expenses will be reimbursed. For overseas applicants we will often conduct interviews by phone or skype. Most decisions about funding are made between February and April, but for UK applicants there is also a possibility to make early offers to outstanding applicants who apply earlier.

Once a student accepts an offer, admission is handled by the Faculty of Science. Evidence must be provided of financial support for fees and subsistence. For most UK students this is satisfied by the award of a studentship

For all other inquiries, please contact:

Dr. Thomas Jordan
School of Mathematics
University Walk
University of Bristol
Bristol BS8 1TW
UK

Telephone: 0117 331 5246
email: thomas.jordan@bris.ac.uk

The fees for full-time postgraduate study are:

British and EU students:	£3,732 (2011-2012)
Other overseas students:	£14200 (2011-2012)

The fees normally change every year in line with inflation. The University believes that overseas students require about £9,000 per annum to live in Bristol.

Overseas students whose native language is not English are required to give evidence of their fluency in English. There are various ways in which this may be done, e.g. in many countries there are offices of the British Council where a test such as the IELTS may be taken leading to a report for the University. The University's normal minimum requirement is 6.5 on IELTS and 600 on TOEFL (old style) and 250 on TOEFL (computerised). However, the Faculty of Science offers a degree of flexibility for postgraduate students in Mathematics and will accept an IELTS -6.0, TOEFL-577(233 computer based) -90(internet based).

11 Closing Dates

There is no fixed closing date for application for M.Sc. or Ph.D. study. However, we strongly urge prospective candidates to apply as early as possible (preferably before **April**) to facilitate the admissions procedure and tenable decisions to be made speedily.

Overseas students should check the webpage
`http://www.bris.ac.uk/studentfunding/overseas_pg/`
for news about sources of funding and deadlines. EU students wishing to apply for funding should check
`http://www.bristol.ac.uk/studentfunding/eu.html`